Nonlinear standing waves in bounded plasmas

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(Received 14 December 2001; revised manuscript recieved 15 July 2002; published 9 October 2002)

An analytical model for nonlinear volume oscillations in a bounded cold plasma is developed. Although here the familiar propagating wave solutions do not exist, exact nonlinear standing waves subject to appropriate boundary conditions can nevertheless be found. The behaviors of the electrons and ions are described self-consistently in terms of Lagrangian variables. The analytical solutions are compared with that from particle-in-cell simulations. Good agreement is found in the regimes of interest.

DOI: 10.1103/PhysRevE.66.046403 PACS number(s): 52.35.Mw, 52.35.Fp, 52.65.Rr

I. INTRODUCTION

Nonlinear cold plasma oscillations is a topic of considerable interest since it is the simplest nonlinear collective phenomenon in plasmas. Theoretical descriptions of such oscillations usually rely on perturbation methods. In certain special cases, exact solutions can also be found. Such solutions are not only useful in interpreting observed phenomena in more detail, they are also needed for verifying new analytical approximations and numerical schemes. A solution describing nonlinear plasma waves in an infinite cold plasma was given by Akhiezer and Lyubarskii [1]. It is a special solution of the cold plasma equations: all variables depend on space and time only through the combination $\xi = t - x/v_{\mathrm{ph}}$, where the phase velocity v_{ph} is constant. Mathematically, the governing equations are then reduced to a much simpler set of ordinary differential equations describing stationary traveling waves. The coordinate ξ is suitable only if the spatial extent is infinite in the x direction. For a bounded system this approach is not useful since the moving coordinate ξ in general cannot describe a given boundary properly. In fact, nonlinear standing surface waves in bounded plasmas are still not well understood, although some progress has been made [2]. In particular, Dawson [3] obtained general solutions for an electron fluid with fixed ions using Lagrangian variables. With this approach, solutions for a bounded plasma can also be obtained [4]. Dawson's solution has since then been generalized [5], but only for a non-self-consistent ion distribution.

Most existing studies of oscillations in bounded plasmas assume that the oscillations take place in a plasma rigidly bounded by a real or virtual wall, and boundary conditions are thus applied at the rigid plasma-wall or plasma vacuum interface. However, in many plasmas, because of their gaseous nature and the electrostatic effects, the plasma boundary is self-consistently determined and almost never rigid. In fact, it is expected that the boundary behavior and the oscillations affect each other. In order to investigate such a scenario, in this paper we present the results of a direct nonperturbative analysis of nonlinear oscillations in a multicomponent cold plasma cloud. The plasma is thus bounded in the region

$$r = \left(\sum_{i=1}^{D} x_i^2 \right)^{1/2} \leqslant R(t),$$

where $D = 1,2$ or 3 for the slab, cylinder or spherical plasma geometry, respectively. In view of eventual applications to an electron-positron plasma, for which the present problem is of special interest [6], we do not invoke the large ion mass approximation [7]. All particles are then treated on the same level in terms of Lagrangian variables. Furthermore, in order that realistic conditions at the boundary (to be defined) can be applied and studied, we allow for self-consistently determined boundary evolution. The latter is important since in applications it often occurs that a physical boundary is not necessarily the plasma boundary, especially when the plasma is not in a steady state. The solutions found are *exact* in the sense that starting from the cold plasma fluid equations no approximation or truncation is made. Exact solutions for nonlinear boundary problems are extremely rare, and the few cases in which they can be found are consequently of great interest. Such solutions usually describe the plasma behavior in a simple manner. They are thus especially suitable as a starting point for understanding the nonlinear behavior of plasmas. We also investigate the relation between divergent cold plasma solutions and the boundary behavior.

II. BASIC EQUATIONS

We shall consider spatially symmetric oscillations, so that the plasma parameters depend on the time t and one spatial variable r. For the slab geometry this does not impose any significant restriction on our model. For the cylindrical or spherical geometry, we shall assume radial symmetry. The plasma is described by the standard cold-fluid model

$$\partial_t n_\alpha + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0, \tag{1}$$

$$\partial_t \mathbf{v}_\alpha + (\mathbf{v}_\alpha \cdot \boldsymbol{\nabla})\mathbf{v}_\alpha = \frac{q_\alpha}{m_\alpha}\mathbf{E}, \tag{2}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 4\pi \sum_\alpha q_\alpha n_\alpha, \tag{3}$$

where the electric field $\mathbf{E}$ is purely electrostatic and α denotes the particle species. Here, $n_\alpha(r,t)$, $\mathbf{v}_\alpha(r,t)$, q_α, and m_α are the particle density, velocity, charge, and mass, respectively.

We now consider the boundary conditions at $r=R(t)$, the boundary of the plasma. For a multicomponent plasma it is then first necessary to define R. For that purpose we introduce an average velocity of the plasma fluid,

$$\mathbf{V} = \frac{\sum_\alpha m_\alpha n_\alpha \mathbf{v}_\alpha}{\sum_\alpha m_\alpha n_\alpha},$$

which is the ratio of the total momentum flux and mass density. Generally, it is close to the velocity of the heavy species. Assuming a free boundary, the evolution of the latter is determined by the normal component of the fluid velocity, i.e., by the radial component V_r for our geometry. Thus

$$\frac{dR(t)}{dt} = V_r(r,t)\big|_{r=R(t)}, \tag{4}$$

where $R(0)=R_0$. We note that this self-consistent plasma-determined boundary condition differs from those of most existing studies of nonlinear plasma oscillations, where a rigid metal or dielectric boundary is assumed.

If the difference between $\mathbf{V}$ and $\mathbf{v}_\alpha$ is small, then the resulting thin layer of charge separation can be replaced by a surface charge at $r=R(t)$. Due to the symmetry there is no surface current, and the surface charge density σ does not depend on the spatial variables, so that $\sigma=\sigma(t)$. It is determined by the equation

$$\frac{d\sigma}{dt} + \frac{D-1}{R}\frac{dR}{dt}\sigma = \sum_\alpha q_\alpha n_\alpha (v_{\alpha r} - V_r)\big|_{r=R(t)}, \tag{5}$$

where the term on the right hand side is responsible for the charge supplied at the boundary by the electric current. The second term on the left-hand side originates from the time-dependent boundary curvature. It is of interest to point out that even if there is no radial current to the boundary (i.e., $\mathbf{v}_\alpha \equiv \mathbf{V}$), $\sigma(t)$ can still vary due to changes of the boundary. Equation (5) results then in $\sigma(t)R(t)^{D-1}=\text{const}$, demonstrating charge conservation.

Finally, the electric field should satisfy the boundary condition

$$E_{\text{out}} - E_{\text{in}} = 4\pi\sigma, \tag{6}$$

where E_{in} and E_{out} are the radial components of the electric field at $r=R(t)$ inside and outside the plasma, respectively. It should be pointed out that if the surface is allowed to deform asymmetrically, nonlinear surface currents will have to be considered.

III. NATURE OF THE SOLUTIONS

Equations (1)–(6) form a closed set. Despite of the symmetry assumed, the set is strongly nonlinear and complex, especially because of the boundary conditions. Nevertheless, it is possible to obtain a class of special solutions for this problem.

The key feature of the special solutions we seek is that the plasma velocity and electric field have a linear spatial dependence, namely

$$n_\alpha = n_\alpha(t), \quad \mathbf{v}_\alpha = \tilde{v}_\alpha(t)\mathbf{r}, \quad \mathbf{E} = \tilde{E}(t)\mathbf{r}, \tag{7}$$

so that the basic equations (1)–(3) are reduced to a set of ordinary differential equations. The reduction is exact in the sense that no approximation (e.g., series expansion or higher harmonics truncation) has been made. The electric field distribution corresponds to the well-known parabolic potential which appears in many laboratory plasma models.

The *Anzätze* (7) have been used earlier for *one-component* systems to describe nonlinear surface and volume waves [8], as well as for oscillations of trapped non-neutral plasmas [9] and gravitating fluids [10]. We now apply it to a multicomponent system. To understand the underlying physics and to simplify the algebra, it is practical to use Lagrangian coordinates. We let $\mathbf{r}_0$ be the initial position of some plasma particle, and assume that the motion of the particle is given by $\mathbf{r}(t)=A_\alpha(t)\mathbf{r}_0$, where the propagator $A_\alpha(t)$ is *identical* for all particles of the same type. The corresponding fluid velocity is then given by Eq. (7), with

$$\tilde{v}_\alpha(t) = \frac{1}{A_\alpha(t)}\frac{dA_\alpha(t)}{dt}$$

and $A_\alpha(0)=1$. The transformation to the variable A_α leads to significant simplification. The continuity equation becomes

$$n_\alpha(t) = \frac{n_{0\alpha}}{A_\alpha(t)^D},$$

with $n_{0\alpha}$ being the initial density. Equation (3) leads to

$$\tilde{E}(t) = \frac{4\pi}{D}\sum_\alpha \frac{q_\alpha n_{0\alpha}}{A_\alpha(t)^D}, \tag{8}$$

so that the Euler equation can be put in the closed form

$$\frac{1}{A_\alpha(t)}\frac{d^2A_\alpha(t)}{dt^2} = \frac{4\pi}{D}\frac{q_\alpha}{m_\alpha}\sum_\beta \frac{q_\beta n_{0\beta}}{A_\beta(t)^D}, \tag{9}$$

which then completely determines the plasma dynamics.

Before proceeding it is first necessary to preclude the possibility that two fluid particles starting from $\mathbf{r}_1$ and $\mathbf{r}_2$, say at $t=0$, intersect at some later time t_1. Since the trajectories of any two particles are given by $A_\alpha(t)\mathbf{r}_1$ and $A_\alpha(t)\mathbf{r}_2$, an intersection is possible only at the origin and it takes place if and only if the propagator $A_\alpha(t_1)$ is identically zero. Only solutions with $A_\alpha(t)>0$ are then physically meaningful in the cold plasma framework.

We now turn to the boundary conditions. It is useful to introduce the quantities

$$\mu(t)=\sum_\alpha \frac{m_\alpha n_{0\alpha}}{A_\alpha(t)^D},\quad \rho(t)=\sum_\alpha \frac{q_\alpha n_{0\alpha}}{A_\alpha(t)^D},$$

which represent the mass and charge densities of the plasma, respectively. The fluid velocity $\mathbf{V}$ and current density $\mathbf{j}$ can then be put into the form

$$\mathbf{V}=-\frac{1}{D}\frac{1}{\mu(t)}\frac{d\mu}{dt}\mathbf{r},\quad \mathbf{j}=\sum_\alpha q_\alpha n_\alpha \mathbf{v}_\alpha=-\frac{1}{D}\frac{d\rho}{dt}\mathbf{r},$$

where both right-hand sides are total derivatives. Equation (4) can thus be integrated to

$$\mu(t)R(t)^D=\text{const},\tag{10}$$

which confirms total mass conservation. Using $R(t)$ and the expression (8) for $\tilde{E}$, we can calculate the electric fields on the plasma boundary

$$E_{\text{in}}=\frac{4\pi}{D}\rho(t)R(t),\quad E_{\text{out}}=0,\tag{11}$$

the latter being valid for any plasma with radial symmetry and overall plasma neutrality. Equation (5) then becomes

$$\frac{d\sigma}{dt}+\frac{D-1}{R(t)}\frac{dR}{dt}\sigma(t)=-\frac{1}{D}\frac{d\rho}{dt}R(t)-\frac{dR}{dt}\rho(t),\tag{12}$$

where Eq. (10) was used to get rid of $\mu(t)$. Equation (12) can be put into the integrable form

$$\frac{d}{dt}\left[R(t)^{D-1}\sigma(t)+\frac{1}{D}R(t)^D\rho(t)\right]=0,$$

which simply verifies charge conservation. Thus, we have

$$\sigma(t)=-\frac{1}{D}\rho(t)R(t),\tag{13}$$

which is a consequence of overall plasma neutrality. From Eqs. (11) and (13) we see that the boundary condition (6) for the electric field is satisfied exactly. The problem is then reduced to solving Eq. (9).

IV. RESULTS

We see that the dynamics of the system is solely determined by Eq. (9). For simplicity we now consider a two-component plasma with equal number of negatively (e) and positively (i) charged particles and $q_e=-q_i$. We have then two second-order equations, namely

$$\frac{d^2A_e(t)}{dt^2}=\frac{\omega_{\text{pe}}^2}{D}\left[\frac{1}{A_e(t)^D}-\frac{1}{A_i(t)^D}\right]A_e(t),\tag{14}$$

$$\frac{d^2A_i(t)}{dt^2}=\frac{\omega_{\text{pi}}^2}{D}\left[\frac{1}{A_i(t)^D}-\frac{1}{A_e(t)^D}\right]A_i(t),\tag{15}$$

where $\omega_{p\alpha}=(4\pi q_\alpha^2 n_{0\alpha}/m_\alpha)^{1/2}$ is the plasma frequency of the species $\alpha=e,i$. Recall that $A_\alpha(0)=1$ and D is the dimension of the system. The equations are valid as long as $A_\alpha(t)>0$, otherwise the trajectories intersect and collapse occurs.

The simplest solution $A_\alpha=1$ corresponds to an equilibrium of the cold plasma cloud. It should be mentioned that the equilibrium exists only because of the neglect of the pressure forces in the governing equations. Generally the plasma tends to expand, forming both a front moving forward and a backward propagating rarefraction wave [11]. Any constant-density cloud will thus be destroyed in a time R_0/c_s, the ratio of the plasma half-size and the sound speed. However, this time scale is large compared with the period of the plasma oscillations studied here.

For small (linear) oscillations around the equilibrium, we readily obtain two characteristic modes. One corresponds to the expected plasma oscillations with the frequency $(\omega_{\text{pe}}^2+\omega_{\text{pi}}^2)^{1/2}$ and the other is a zero frequency mode. The latter is confirmed by the existence of a solution with $A_e(t)=A_i(t)$, and both functions being linear in t. The solution describes plasma expansion with constant speed. That is, the cloud is unstable even in the pressureless model.

Our results are valid for arbitrary mass ratio. Nontrivial new results can also be obtained for electron-ion plasmas. In this case, we consider oscillations in the expanding plasma cloud and neglect terms of order m_e/m_i. We have then

$$A_i=1+\frac{V_0 t}{R_0},$$

where the constant V_0 is the fluid velocity. For $V_0<0$ the ion trajectories inevitably intersect, resulting in collapse of the solution, and for $V_0>0$ the plasma expands. Note that despite the small mass ratio, the ions do move and therefore affect the electron dynamics. The latter is described by

$$D\frac{d^2A_e}{d\tau^2}+\frac{1}{(1+\tau/k)^D}A_e(t)=\frac{1}{A_e(t)^{D-1}},\tag{16}$$

where $\tau=\omega_{\text{pe}}t$ is a dimensionless time, and the parameter

$$k=\frac{\omega_{\text{pe}}R_0}{V_0}$$

is the ratio of the characteristic ion and electron time scales in this problem. For $V_0=0$, Eq. (16) was considered by Dawson [3], who obtained solutions for fixed ions.

We now take into account the ion dynamics. Equation (16) is linear if $D=1$ (e.g., in the slab geometry). In this case we can directly solve it to obtain

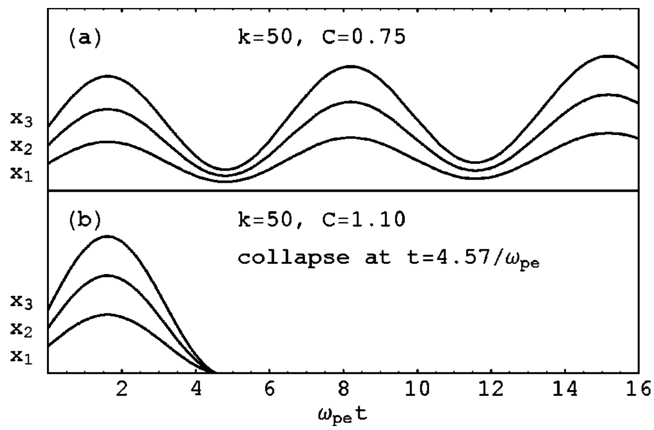

FIG. 1. Coordinate (arbitrary units) versus time for neighboring electrons: (a) regular behavior; (b) collapsing solution.

$$A_e = A_i + \sqrt{A_i}[c_1 J_1(2k\sqrt{A_i}) + c_2 Y_1(2k\sqrt{A_i})],$$

where J and Y are the Bessel functions of the first and second kind. The constants $c_{1,2}$ should be chosen to satisfy the condition $A_e(0)=1$.

In practice, the ratio k is usually large. The expression for A_e can then be simplified to

$$A_e = A_i + C\sqrt[4]{A_i}\sin[2k(\sqrt{A_i}-1)], \tag{17}$$

where C is a constant. The other plasma parameters can thus be readily obtained. For $V_0 \to 0$ our solution becomes $A_e = 1 + C\sin\omega_{pe}t$, which is similar to that given earlier [4]. Equation (17) shows that the electron oscillations are both aperiodic and modulated by the ion motion.

Let us now consider the possibility of collapse. It is clear from Eq. (17) that A_e can change sign, resulting in trajectory intersection. Such a collapse occurs if C exceeds some critical value $C^*(k) \approx 1$. Figure 1 shows the trajectories of neighboring electrons for $k=50$ and two values of C. The regular trajectories for $C<C^*$ are replaced by the intersecting ones for $C>C^*$. In the latter case our cold plasma solution becomes invalid after a finite time.

For $D=2,3$, Eq. (16) is both nonlinear and nonautonomic. Analytical solutions can only be obtained for either small oscillations or nonlinear ones with $V_0=0$. More general cases must be obtained numerically. All solutions show that the nonlinear oscillations are modulated and aperiodic. Furthermore, for time scales comparable with $1/\omega_{pi}$, the terms of order m_e/m_i cannot be neglected. Equations (14) and (15) can no longer be separated, and must be solved numerically.

An interesting case is the electron-positron plasma with $m_e = m_p$, so that the plasma frequencies in Eqs. (14) and (15) are equal. The motion of the positrons must thus be taken into account from the very beginning. Numerical solutions for this case are included in the next section.

V. SIMULATIONS

In this section we obtain numerical solutions of Eqs. (14) and (15) for self-consistently evolving electron-ion and electron-positron plasma clouds. To see how these special solutions might appear in reality, we have also performed particle-in-cell (PIC) simulations of the corresponding problem in one dimension. We followed 10^4 electrons and as many ions or positrons. Due to symmetry only the region $x>0$ needs to be considered in the computation. Each particle represents a layer with a surface charge q, and n_0 is the number of such layers per unit length at $t=0$. The electric field is normalized by $4\pi q n_0 R_0$, the time by $1/\omega_{pe}$, and the length by R_0.

Initially the electrons and ions (positrons) were distributed with constant and equal densities, and a sharp boundary at $x=R_0$ was assumed. To allow for plasma expansion the total area of the computation was taken to be considerably larger than R_0. The boundary at $x=0$ representing the plasma center was taken to be both absorbing and reemitting, and the boundary at the vacuum side was assumed to be absorbing only. The particles were initialized with a Maxwellian velocity distribution such that $R_0 = 200\lambda_{de}$, where λ_{de} is the electron Debye length. The electric field was computed by integrating the charge density starting from $E=0$ at $x=0$. In the simulation the plasma was warm, so that the validity of our analytical cold plasma model can be studied. The temperature ratio was 25 for an electron-ion plasma, and unity for an electron-positron plasma.

In general, the plasma slowly expands, forming both a front and a rarefraction wave. Oscillations were initialized by forcing the particles to perform additional (nonthermal) motion with initial velocities proportional to their positions. Thus, the initial regular particle velocity inside the plasma ranges from 0 at $x=0$ to $R_0 dA_\alpha/dt|_{t=0}$ at $x=R_0$. The magnitude $dA_\alpha/dt|_{t=0}$ was taken to be small enough to avoid large velocities of the boundary particles. In most cases a linear approximation to Eqs. (14) and (15) should thus be sufficient.

First we consider small oscillations. The velocities of the boundary electrons and ions were $2v_{te}$ (thermal velocity) and $0.9c_s$, respectively, and the mass ratio was $m_i/m_e = 900$. To obtain the density we evaluated the fluid velocity V_0 from the initial conditions, and took the region $x<0.8(R_0+V_0t)$ as the uniform-density center of the plasma. The corresponding values of $n_\alpha(t)$ were compared with that predicted by Eqs. (14) and (15) as shown for the electrons in Fig. 2. The

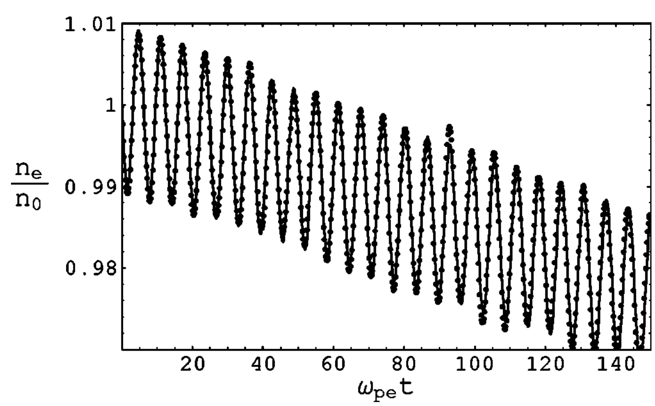

FIG. 2. Electron density versus time for small perturbations. The simulation result (points) agrees well with that of theory (solid line).

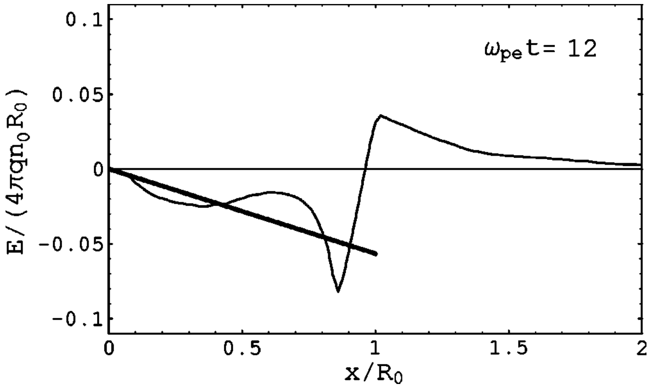

FIG. 3. Typical behavior of the electric field for large perturbations. The theory (thick line) correctly matches the linear part of the electric field inside the plasma.

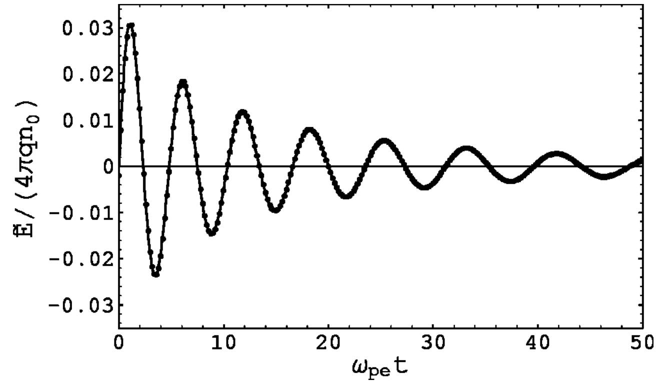

FIG. 5. $\tilde{E}$ versus time for a rapidly expanding electron-positron plasma. Simulation (points) is in good agreement with theory (solid line).

density decrease due to the plasma expansion can clearly be observed. The theory agrees well with the simulation for several tens of oscillation periods.

Next we consider the case of strong perturbations: the velocity of the boundary electrons was taken to be $20v_{te}$, $m_i/m_e = 2500$, and $dA_i/dt|_{t=0} = 0$. Figure 3 shows the typical spatial behavior of the electric field from the theory and simulation at one particular instant ($\omega_{pe}t = 12$).

We see that Eqs. (14) and (15) correctly describe the behavior of the electric field inside the evolving plasma. We have also obtained the best linear fit of the electric field at the uniform-density center of the plasma and compared it with the theoretical value of $\tilde{E}$, as shown in Fig. 4. Good agreement occurs for several periods. The simulation shows a steady modulation of the electric field. This asymmetric modulation may be attributed to the mixing phenomena due to the nonuniform density at the plasma boundary [3].

We have also simulated an electron-positron plasma with $m_e = m_p$. The plasma also expands naturally, but with a rate larger than that of the electron-ion plasma. The size of the boundary layer quickly becomes comparable to that of the uniform-density region. Oscillations were initialized in the expanding layer. The boundary electrons and positrons were given the initial velocities $20v_{te}$ and $10v_{te}$, respectively. The size of the uniform-density region is again approximately given by $R_0 + V_0 t$. The theory agrees well with the simulation, as shown in Fig. 5 for the electric field. The plasma density decreases considerably within the simulation period because of the fast expansion, the rate of the decrease is also in a good agreement with the theory.

All the above solutions correspond to cases with $V_0 > 0$, e.g., the plasma expands. If initially one has $V_0 < 0$, the solution of Eqs. (14) and (15) shows collapse behavior. That is, $A_\alpha(t)$ tends to zero for one of the species, and the density tends to infinity. The collapse is related to the trajectory intersection at $x = 0$.

Collapse rapidly occurs for very strong perturbations with $dA_e/dt \sim \omega_{pe}$. For instance, let us consider an electron-positron plasma with $dA_e/dt|_{t=0} = -0.5\omega_{pe}$ and $dA_p/dt|_{t=0} = 0$. We used 10^5 particles for the simulation. A typical electron and positron density evolution at one particular instant ($\omega_{pe}t = 2.2$) is shown in Fig. 6.

The analytical theory is now applicable only to a rapidly shrinking region with approximately constant density near $x = 0$. We determined $n_\alpha(t)$ for $x < 2\lambda_{de}$ and compared it with the theory. The positron density presented in Fig. 7 clearly shows tendency of collapse, but the singularity predicted by the theory was not realized in the simulation. Note

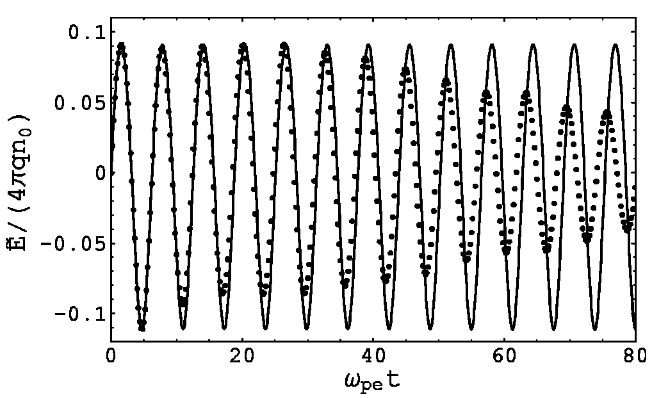

FIG. 4. $\tilde{E}$ versus time obtained from the simulation (points) and theory (solid line). The plasma is significantly perturbed.

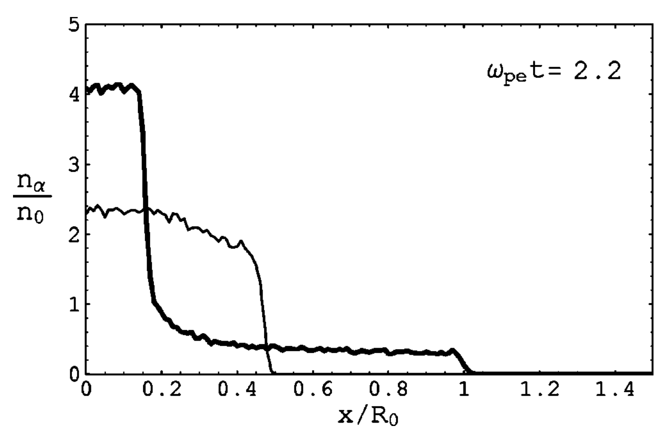

FIG. 6. Electron (thin line) and positron (thick line) density versus position for a collapsing solution. The theory describes only the small constant-density region near the plasma center.

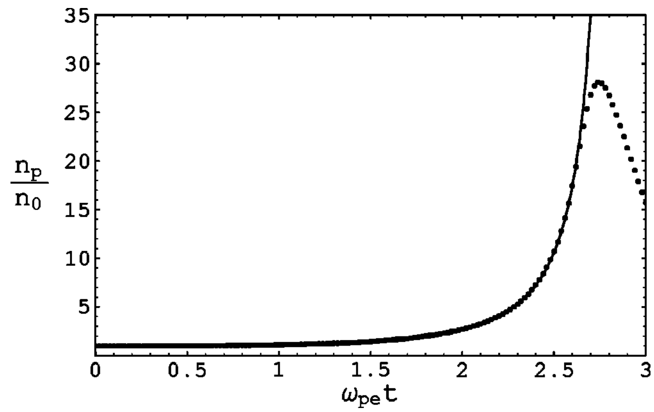

FIG. 7. Positron density from simulation (points) and from theory (solid line) versus time for a collapsing solution.

that collapse first occurs for the positrons, although only the electrons were initially perturbed.

VI. CONCLUSION

In this paper we have considered the nonlinear evolution of a multicomponent plasma cloud. The plasma boundary and oscillations are allowed to evolve naturally. Analytical solutions for cold plasma oscillations are obtained. The solutions are exact in the sense that no approximation (e.g., series expansion or higher harmonics truncation) has been made. The evolution is subject to realistic boundary conditions. In contrast to the earlier works on bounded plasmas where a fixed physical boundary is usually assumed, we allow for the fact that the plasma boundary may not be the original one. In fact, our results are also applicable to isolated plasma clouds in vacuum. Furthermore, we do not apply the two-time-scale approach and do not use the large ions mass approximation. All particles are treated at the same level in terms of Lagrangian variables. Thus equations obtained are also applicable to electron-positron plasmas. The solutions show that the plasma oscillations and boundary evolution are strongly related. The results agree well with that of PIC simulations for the regimes of interest.

ACKNOWLEDGMENT

One of the authors (Sh.A.) would like to thank the Humboldt Foundation for financial support.

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